ANALYTICAL AND EXPERIMENTAL STUDY OF MIXED CONVECTION IN TUBE BUNDLES WITH A LONGITUDINAL FLOW

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A solution is given to the heat problem of mixed convection in vertical tube bundles with a relative spacing s/d = 1.2 and 1.4. The results have been checked out experimentally with a forced flow of liquid sodium.

Liquid-metal heat carriers flow at low velocities in many components of nuclear power plants both under steady-state design conditions and during transients.

Large temperature gradients which appear under these conditions may have an appreciable effect on the heat transfer. At small values of the Peclet number, characteristic of such flow modes, the effect of turbulent transfer is small. In view of this, an analytical study becomes much simpler and can be done with satisfactory engineering accuracy.

For solving the problem of mixed convection in a vertical tube bundle under hydrodynamically and thermally stable conditions, the authors have made certain simplifying assumptions: not only that the flow is hydrodynamically and thermally stable but also that the physical properties of the medium (except its density) are independent of the temperature, the density is a linear function of the temperature, and the temperature gradient along the channel is constant. Under these stipulations, the equation of heat transfer and the equation of hydrodynamics become

$$w_x \frac{\partial t}{\partial x} = a\Delta t, \tag{1}$$

$$-\rho g - \frac{\partial P}{\partial x} + \mu \Delta w_x = 0, \tag{2}$$

where $\partial t/\partial x = A = \text{const}$; $\rho = \rho_0[1-\beta(t-t_c)]$.

If one considers bundles with a wide relative spacing only ($s/d \gtrsim 1.4$), then the method of an equivalent annulus will make these equations one-dimensional:

$$w_x \frac{dt}{dx} = a\frac{1}{r}\cdot\frac{d}{dr} r \frac{dt}{dr}, \tag{1a}$$

$$-\rho g - \frac{dP}{dx} + \mu\frac{1}{r}\cdot\frac{d}{dr} r \frac{dw_x}{dr} = 0. \tag{2a}$$

Differentiating the last equation and eliminating t will yield the fourth-order equation:

$$\frac{1}{\xi}\cdot\frac{d}{d\xi}\,\xi\frac{d}{d\xi}\left(\frac{1}{\xi}\cdot\frac{d}{d\xi}\,\xi\frac{d\omega}{d\xi}\right) - k^4\omega = 0, \tag{3}$$

where

$$\xi = \frac{r}{R};\quad \omega = \frac{w_x}{\overline{w}};\quad k^4 = \text{Ra} = \frac{g\beta A R^4}{\nu a},$$

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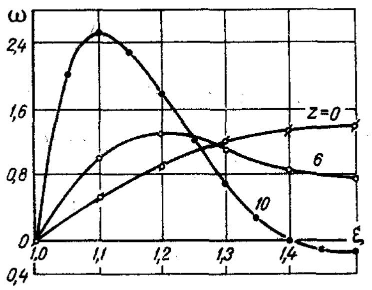

Fig. 1

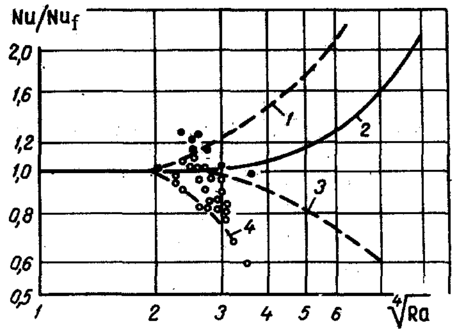

Fig. 2

Fig. 1. Calculation according to Eq. (6), for ascending flow; $z = \sqrt[4]{Ra}$.

Fig. 2. Test data on heat transfer with natural convection in the tube bundle: ascending flow (1, 2); descending flow (3, 4). Calculations according to Eq. (7) for a tube bundle with s/d = 1.4 with ascending flow (2) and descending flow (3). Analogous data for the tube in [1] (1, 4).

with the boundary conditions

$$\begin{gathered} \xi = 1; \quad \omega = 0, \\ \xi = \xi^*; \quad \frac{d\omega}{d\xi} = 0, \\ \frac{d}{d\xi}\left[\frac{1}{\xi}\cdot\frac{d}{d\xi}\xi\frac{d\omega}{d\xi}\right] = 0. \end{gathered} \tag{4}$$

Here ξ^* represents the outside boundary of the equivalent annulus, and the last equation satisfies the condition of temperature field symmetry;

$$\int_1^{\xi^*} \omega\xi d\xi = \frac{\xi^{*2} - 1}{2} \tag{5}$$

is the integral condition.

The general solution to Eq. (3) appears in two forms depending on the algebraic sign of $Ra = k^4$.

When natural convection and forced convection ($Ra < 0$) concur (at the wall), then the solution for the velocity field is

$$\omega = A\,\mathrm{ber}(k\xi) + B\,\mathrm{bei}(k\xi) + C\,\mathrm{ker}(k\xi) + D\,\mathrm{kei}(k\xi). \tag{6}$$

Functions berx, beix, kerx, and keix are Thompson functions. They are proportional, respectively, to the real and the imaginary parts of first-kind and second-kind Bessel functions of the argument $x\sqrt{j}$. Constants A, B, C, and D are determined from conditions (4) and (5).

The values of ω calculated according to Eq. (6) for $\xi^* = 1.5$ are shown in Fig. 1.

The Nusselt number was calculated according to the formula for an equivalent annulus with the modified Lyon integral:

$$Nu = \left[\frac{2}{(\xi^{*2} - 1)^3}\int_1^{\xi^*} \frac{\left(\int_1^{\xi^*} \omega\xi d\xi\right)^2}{\xi} d\xi\right]^{-1} \tag{7}$$

The results of numerical integration for $\xi^* = 1.5$ are shown in Fig. 2.

Analogous calculations were made for $Ra > 0$ (descending flow). Here the general solution for the velocity field is

$$\omega = AJ_0(k\xi) + BY_0(k\xi) + CI_0(k\xi) + Dk_0(k\xi). \tag{8}$$

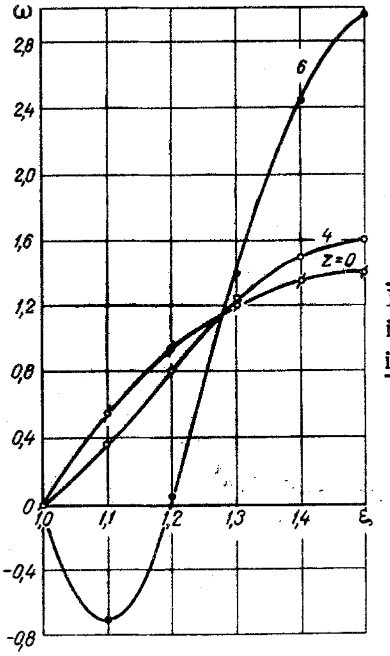

Fig. 3. Calculation according to Eq. (8), for descending flow.

The results of calculations by formulas (7) and (8) for descending flow with $\xi^* = 1.5$ are shown in Figs. 2 and 3.

For comparison, in Fig. 2 are also shown the calculations in [1] pertaining to the effect of natural convection during descending and ascending flow through a tube.

The effect of natural convection on the heat transfer was verified experimentally in tube bundles with a relative spacing s/d = 1.4 and 1.2 during ascending and descending flow of sodium.

The tests were performed in a closed circulation system with an electromagnetic pump. The geometrical characteristics of the test bundles are given in Table 1.

The bundles consisted of 7 tubes and a jacket with expellers. The same calorimeter with movable thermocouples was used in the tests with s/d = 1.4 and s/s = 1.2 bundles, this calorimeter being placed at the center of a bundle.

Each bundle was tested with the heat carrier flowing in either of the two directions: downward or upward (the active segments of the system were stabilized vertically). In these tests the authors measured the temperature of the calorimeter wall, the temperature of sodium at the entrance to and at the exit from the bundle as well in the center cells at a distance 50 mm away from the heating zone, the flow rate of sodium, and the electric power drawn by the heaters. The heat-transfer coefficients were defined in terms of the dimensionless number

$$\mathrm{Nu} = \frac{\alpha d}{\lambda}.$$

The temperature difference defining the heat-transfer coefficient for the thermal stabilization zone ($\alpha = q/\Delta t$) was calculated by two methods. First it was determined from the difference between the temperature reading at the wall and the temperature at the center of cells adjoining the calorimeter (this temperature being recorded on a coordinate plotter). This difference was corrected for the temperature drop across the calorimeter wall (the calorimeter was made of copper). In our case the temperature difference was defined in terms of the maximum temperature drop. For that reason, it included the correction equal to the average between one for jet flow and one for laminar flow. These corrections, in turn, were calculated according to formulas by Sparrow and Loeffler [2, 3] for the temperature field and the mean temperature differences. For bundles with a relative spacing s/d = 1.4, $\Delta t_{max}/\Delta t_{mean} = 1.4$ in the case of a jet stream through the tubes and $\Delta t_{max}/\Delta t_{mean} = 1.2$ in the case of a laminar stream through the tubes. In the second method of calculating the temperature difference for the heat-transfer coefficient, a straight line corresponding to the mean-over-the-mass stream temperature was drawn parallel to the straight line representing the wall temperature past the stabilization zone. It must be noted that, inasmuch as the tests were performed at small values of the Peclet number, the actual stream temperature was probably

TABLE 1. Geometrical Characteristics of Tube Bundles

Bundle	Geometrical parameters					
	inside diameter of jacket, mm	diameter of tube, mm	active cross section, mm^2	length of heating zone, mm	hydraulic diameter of cell, d_∞, mm	hydraulic diameter of bundle, d_h, mm
$s/d = 1,4$	107	22	5240	800	25,5	25,5
$s/d = 1,2$	88	22	2581	800	12,9	12,9

somewhat higher because of heat leakage from exit to entrance. By evaluating the data in the form shown earlier, we obtained a Nusselt number Nu* form which the actual Nusselt number could be calculated according to the formula in [4]:

$$\mathrm{Nu} = \frac{\mathrm{Nu}^*}{1 - 4\,\dfrac{\mathrm{Nu}^*}{\mathrm{Pe}^2}\,(1+\chi)}.$$

The first method of determining the temperature difference did not require such a procedure, because the stream temperature was measured there directly in the heating zone.

The minimum value of the Peclet number in these tests was Pe ~ 20.

The test data for the bundle with s/d = 1.4 are shown in Fig. 2. Analogous results were obtained also for bundles with s/d = 1.2.

The Nusselt number is almost constant here, inasmuch as Pe < 100, and so along the ordinate axis we have plotted the ratio $\mathrm{Nu}/\mathrm{Nu}_f$ (Nu_f denoting the Nusselt number for purely forced convection). A comparison between test points and calculated curves indicates that Nu differs from Nu_f by more than would be expected theoretically. This can, apparently, be explained by stray measurement errors and, especially, by mutual effects between the center cell and the peripheral cells, which may cause the temperature curves for the center cell to be nonlinear. The distinct segregation of test points corresponding to the two directions of flow, respectively, however, is certainly attributable to natural convection. It is interesting to note that the test points lie closer to the curves for a circular tube. On the other hand, quite clearly, in calculations by the equivalent-annulus method the effect of natural convection appears somewhat reduced because of the reduced maximum transverse dimension of the channel. It may be assumed that the actual curve lies somewhere between the curve for a bundle and the curve for a circular tube (Fig. 2).

Earlier in [5] the authors have numerically solved the problem of heat transfer by mixed convection for a tube bundle with a relative spacing s/d = 1, with various sheath thicknesses and with various tube radii for various values of $\sqrt[4]{\mathrm{Ra}}$. The calculations, made by the boundary collocation method, have shown that in a close packed bundle even at high values of $\sqrt[4]{\mathrm{Ra}}$ (up to 7) the temperature nonuniformity is not very different from the temperature nonuniformity under purely forced convection. A comparison between data for bundles with s/d = 1.4 and 1.2 shows a diminishing, though not very sharply, effect of natural convection here. Our limited test facilities did not make it possible to cover a wide range of $\sqrt[4]{\mathrm{Ra}}$ values, because increasing the diameter of tubes would require long active zones, and yet the longitudinal temperature gradients were already large enough under existing test conditions.

Nevertheless, our tests covered the entire practical range of the Rayleigh number and provided sufficient data to suggest formulas for heat transfer with natural convection.

When natural and forced convection occur in the same direction, then the heat transfer may be calculated by formulas for purely forced convection: some surface margin will then become available. When natural and forced convection occur in opposite directions, then the reduction of heat transfer can be estimated by the formulas for a circular tube. The calculated reduction of heat transfer will also be somewhat too high.

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